Probability

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q1.

Assertion (A): Two coins are tossed simultaneously. The probability of a getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R): Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

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Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q2.

Assertion (A): Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely. If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively, then $P(E/F) = \frac{1}{4}$ and $P(F/E) = \frac{1}{13}$.

Reason (R): E **and** F **are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.**

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q3.

Assertion (A): Let E **and** F **be events associated with the sample space** S **of an experiment. Then, we have** $P(S/F) = P(F/F) = 1$.

Reason (R): If A **and** B **are any two events associated with the sample space** S **and** F **is an event associated with** S **such that** $P(F) \neq 0$**, then** $P((A \cup B)/F) = P(A/F) + P(B/F) - P((A \cap B)/F)$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q4.

Let A **and** B **be two events associated with an experiment such that** $P(A \cap B) = P(A)\,P(B)$.

Assertion (A): $P(A/B) = P(A)$ **and** $P(B/A) = P(B)$

Reason (R): $P(A \cup B) = P(A) + P(B)$

Answer : (c) Assertion (A) is true but Reason (R) is false

Q5.

Let $H_1, H_2, \ldots\ldots\ldots, H_n$ **be mutually exclusive and exhaustive events with** $P(H_i) > 0$, $i = 1, 2, \ldots, n$.

Let E **be any other event with** $0 < P(E) < 1$

Assertion (A): $P(H_i / E) > P(E / H_i) \times P(H_i)$ **for** $i = 1, 2, \ldots\ldots, n$

Reason (R): $\sum_{i=1}^{n} P(H_i) = 1$

Answer : (d) Assertion (A) is false but Reason (R) is true

Q6. Assertion (A): An urn contains 5 red and 5 black balls. A ball is drawn at random; its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. Then, the probability that the second ball is red is 1/2.

Reason (R): A bag contains 4 red and 4 black balls; another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Then, the probability that the ball is drawn from the first bag is 2/3.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q7. Assertion (A): The mean of a random variable X is also called the expectation of X, denoted by E(X).

Reason (R): The mean or expectation of a random variable X is not sum of the probabilities of all possible values of X by their respective probabilities.

Answer : (c) Assertion (A) is true but Reason (R) is false

Q8. Assertion (A): The mean number of heads in three tosses of a fair coin is 1.5.

Reason (R): Two dice are thrown simultaneously. If X denotes the number of sixes, the expectation of X is 1/3.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Assertion (A): Let A and B be two events such that $P(A) = \frac{1}{5}$, while $P(A \text{ or } B) = \frac{1}{2}$. Let $P(B) = P$, then for $P = \frac{3}{8}$, A and B independent.

Reason (R) : For independent events,

$$\begin{aligned} P(A \cap B) &= P(A)\,P(B) \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A)\,P(B) \\ &= \frac{1}{5} + P - \left(\frac{1}{5}\right)P \end{aligned}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{5} + \frac{4}{5}P$$

$$\Rightarrow \quad P = \frac{3}{8}.$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Assertion (A) : If A and B are two mutually exclusive events with $P(\bar{A}) = \frac{5}{6}$ and $P(B) = \frac{1}{3}$. Then $P(A/\bar{B})$ is equal to $\frac{1}{4}$.

Reason (R) : If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.6$ and $P(A|B) = 0.2$ then the value of $P(A|\overline{B})$ is 0.2.

Ans. Option (B) is correct.

Explanation: Assertion (A) is correct.

$$P(A|\overline{B}) = \frac{P(A\cap\overline{B})}{P(\overline{B})}$$

$$P(A|\overline{B}) = \frac{P(A)}{P(\overline{B})}$$

[since, given A and B are two mutually exclusive events]

$$P\left(\frac{A}{\overline{B}}\right) = \frac{\left(1-\frac{5}{6}\right).}{\left(1-\frac{1}{3}\right)} = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{4}$$

Reason (R) is also correct.

For independent events,

$$P(A|\overline{B}) = P(A) = 0.2.$$

Assertion (A) : Let A and B be two events such that the occurrence of A implies occurrence of B, but not vice-versa, then the correct relation between $P(A)$ and $P(B)$ is $P(B) \geq P(A)$.

Reason (R) : Here, according to the given statement

$$A \subseteq B$$

$$P(B) = P(A \cup (A\cap B)) \qquad (\because A\cap B = A)$$

$$= P(A) + P(A\cap B)$$

Therefore, $P(B) \geq P(A)$

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Assertion (A) : If $A\subset B$ and $B\subset A$ then, $P(A) = P(B)$.

Reason (R) : If $A\subset B$ then $P(\overline{A}) \leq P(\overline{B})$.

Ans. Option (C) is correct.

Explanation : Assertion (A) is correct.

$A\subset B$ and $B\subset A \Rightarrow A = B$

Hence, $P(A) = P(B)$.

But (R) is wrong.

$$A\subset B \Rightarrow \overline{B}\subset\overline{A}$$

Therefore, $P(\overline{A}) \geq P(\overline{B})$

Assertion (A) : The probability of an impossible event is 1.

Reason (R) : If A is a perfect subset of B and $P(A) < P(B)$, then $P(B-A)$ is equal to $P(B)-P(A)$.

Ans. Option (D) is correct.

Explanation : Assertion (A) is wrong.

If the probability of an event is 0, then it is called as an impossible event.

But Reason (R) is correct.

From Basic Theorem of Probability,

$P(B-A) = P(B) - P(A)$, this is true only if the condition given in the question is true.

Assertion (A) : If $A = A_1 \cup A_2 ...\cup A_n$, where $A_1 ... A_n$ are mutually exclusive events then $\sum_{i=1}^{n} P(A_i) = P(A)$

Reason (R) :

Given, $A = A_1 \cup A_2 ... \cup A_n$

Since $A_1 ... A_n$ are mutually exclusive

$$P(A) = P(A_1) + P(A_2) + ... + P(A_n)$$

Therefore $P(A) = \sum_{i=1}^{n} P(A_i)$

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).